EXHIBIT 27

UNITED STATES DISTRICT COURT
NORTHERN DISTRICT OF ILLINOIS
EASTERN DIVISION

ANDREW CORZO, SIA HENRY, ALEXANDER LEO-GUERRA, MICHAEL MAERLENDER, BRANDON PIYEVSKY, BENJAMIN SHUMATE, BRITTANY TATIANA WEAVER, and CAMERON WILLIAMS individually and on behalf of all others similarly situated,

Plaintiffs,

v.

BROWN UNIVERSITY, CALIFORNIA INSTITUTE OF TECHNOLOGY, UNIVERSITY OF CHICAGO, THE TRUSTEES OF COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK, CORNELL UNIVERSITY, TRUSTEES OF DARTMOUTH COLLEGE, DUKE UNIVERSITY, EMORY UNIVERSITY, GEORGETOWN UNIVERSITY, THE JOHNS HOPKINS UNIVERSITY, MASSACHUSETTS INSTITUTE OF TECHNOLOGY, NORTHWESTERN UNIVERSITY, UNIVERSITY OF NOTRE DAME DU LAC, THE TRUSTEES OF THE UNIVERSITY OF PENNSYLVANIA, WILLIAM MARSH RICE UNIVERSITY, VANDERBILT UNIVERSITY, and YALE UNIVERSITY,

Defendants.

Case No. 22-cv-00125

EXPERT REPORT OF LAUREN J. STIROH, PH.D.

August 7, 2024

Appendix 1

1. Dr. Singer uses his flawed regression model to perform an analysis that he argues can demonstrate common impact. Even ignoring the flaws and shortcomings of his underlying regression model which makes the model inappropriate even to estimate average impact, Dr. Singer's in-sample analysis is incapable of demonstrating that nearly all Class members paid an inflated price. Dr. Singer can only reach the conclusion that substantially all proposed Class members were impacted by incorrectly using the statistical predictions from his regression model. As I explain, this analysis is not consistent with economic theory because it conflates unexplained variation in EIPs with the impact, if any, of the Challenged Conduct. Dr. Singer purports to use an "in-sample" prediction methodology to establish class-wide impact.[1] Dr. Singer claims that this methodology allows him to identify specific individual student-school-year observations that were impacted by the Challenged Conduct.[2] As I demonstrate below, this methodology is fundamentally flawed. A regression model, such as the one employed by Dr. Singer, is only an approximation of the relationships between the explanatory variables included in the model and the outcome variable, and Dr. Singer's model does not fully explain the EIP paid by each proposed Class member in each year.[3] By claiming that he can use his average overcharge regression results to precisely identify which student-school-year observations were impacted, Dr. Singer ignores the inherent variation in his regression and conflates the variation in outcomes unexplained by his model with the estimated average impact from his analysis.

2. As previously discussed, Dr. Singer uses a regression model to estimate the statistical relationship between EIPs and other variables included in his model. Dr. Singer's supposed standard analysis to demonstrate class-wide impact starts with the results of his regression model.[4] Dr. Singer's econometric model can be expressed by the following equation, where EIP_{ist} represents the EIP paid by student i at school s in time t.[5]

$$EIP_{ist} = \alpha + \beta Conduct_{st} + \Sigma_X Controls_{ist}^X + \varepsilon_{ist}$$

3. After running a regression model, fitted values of the dependent variable (in this case $\widehat{EIP_{ist}}$) can be obtained as the linear combination of the estimated coefficients ($\hat{\alpha}$, $\hat{\beta}$, and the estimated coefficients on the control variables) and the values of the independent variables for the particular student-school-year observation.

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1 Singer Amended Report, ¶ 255.

2 Singer Amended Report, ¶¶ 256, 258.

3 William H. Greene, *Econometric Analysis* (6th Edition) (Pearson/Prentice Hall, 2007), p. 9 ("The term ε is a random **disturbance**, so named because it 'disturbs' an otherwise stable relationship. The disturbance arises for several reasons, primarily because we cannot hope to capture every influence on an economic variable in a model, no matter how elaborate.") (emphasis in original).

4 Singer Amended Report, ¶ 255 ("This econometric method uses the output of my regression analyses to compare the Effective Institutional Price that students paid to what they would have paid 'but-for' the Challenged Conduct.").

5 Singer Amended Report, ¶ 238.

$$\widehat{EIP_{ist}} = \hat{\alpha} + \hat{\beta} Conduct_{ist} + \Sigma_X \widehat{Controls}^X_{ist} \quad (fitted\ EIP)$$

4. By subtracting the fitted value of EIP ($\widehat{EIP_{ist}}$) from the actual, observed EIP for each particular student-school-year observation (EIP_{ist}), you obtain the regression residual (r_{ist}).[6] This residual informs how far off the actual value for a particular observation is from the value predicted by the estimated regression model. While multiple regression models are constructed so that these residuals are, on average, equal to zero, this does not mean that the residual is zero for all observations.[7]

$$EIP_{ist} - \widehat{EIP_{ist}} = r_{ist} \quad (residual)$$

5. Dr. Singer's in-sample methodology relies upon comparing actual, observed EIPs with EIPs he asserts would have prevailed in the but-for world absent the Challenged Conduct. To obtain these predicted but-for EIPs (represented as $\widehat{EIP^{bf}_{ist}}$), Dr. Singer uses all of the estimated coefficients from his model except the conduct coefficient to predict a price for each student-school-year observation. These estimated coefficients represent the average estimated relationship between the control variables and prices across all observations in the regression model (*i.e.*, not the effect of the control variables on EIPs at the individual level).[8]

$$\widehat{EIP^{bf}_{ist}} = \hat{\alpha} + \Sigma_X \widehat{Controls}^X_{ist} \quad (predicted\ but-for\ EIP)$$

6. Dr. Singer then takes the difference between actual prices and what his model predicts would be but-for prices for each observation (this difference is depicted in the expression below). He concludes by claiming that any positive difference between these two values is evidence that a particular student-school-year observation was impacted by the alleged conspiracy.[9]

$$EIP_{ist} - \widehat{EIP^{bf}_{ist}}$$

7. The fundamental flaw of Dr. Singer's approach is that, even assuming his regression is well-specified and that he has estimated properly the average effect of the conduct and all of the additional control variables included in his model, the student-school-year level differences he examines to "identify" common impact conflate both the residual values of his regression (r_{ist}) and the average effect, if any, of the conduct ($\hat{\beta}$). Through simple algebra and by the definitions of the regression components used in Dr. Singer's methodology, I show that the difference Dr. Singer analyzes ($EIP_{ist} - \widehat{EIP^{bf}_{ist}}$) can

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6 Jeffrey M. Wooldridge, *Introductory Econometrics: A Modern Approach* (7th Edition) (Boston: Cengage, 2020) ("Wooldridge (2020)"), pp. 27-28, 32-33, 74.

7 Wooldridge (2020), p. 74 ("Normally, the actual value for y_i for any observation *i* will not equal the predicted value, $\hat{y}_i$: OLS minimizes the *average* squared prediction error, which says nothing about the prediction error for any particular observation.").

8 Wooldridge (2020), p. 72.

9 Singer Amended Report, ¶ 258.

equivalently be expressed as the sum of the residual for each individual observation from Dr. Singer's regression and the average estimated conduct parameter ($r_{ist} + \hat{\beta} Conduct_{ist}$).

$$EIP_{ist} - \widehat{EIP_{ist}^{bf}} = EIP_{ist} - \left[\widehat{EIP_{ist}} - \hat{\beta} Conduct_{ist}\right]$$

$$= r_{ist} + \hat{\beta} Conduct_{ist}$$

8. The issue with Dr. Singer's methodology is that any variation in outcomes unexplained by the regression model is conflated with the average effect, if any, from the conduct. Indeed, in a well-specified linear regression model, roughly half the fitted values of a variable will be above the observed values and half will be below.[10] Assuming that all proposed Class members attended Defendant schools for four years (and all four years were in the participation periods), then even if the estimated effect of the conduct were zero, using the "in-sample prediction methodology" suggested by Dr. Singer in this hypothetical setting would be expected to flag 93.75 percent of proposed Class members as being "impacted."[11] Such a flawed method is simply incapable of providing any direction as to whether average impact, even if properly estimated, was class-wide.

9. Another way to illustrate the error with Dr. Singer's methodology is to review the estimated residuals from his regression model, their size and variation, and their connection with Dr. Singer's approach of flagging individual observations as impacted or not impacted. The estimated residual for each student-school-year observation is obtained from taking the difference of actual EIP (EIP_{ist}) and the fitted EIP ($\widehat{EIP_{ist}}$) from Dr. Singer's regression model including the average, estimated conduct coefficient. In **Figure A.1** below, I present a scatterplot of residuals from observations of students at Cornell in academic year 2022-2023.[12] Dr. Singer's methodology mechanically flags any observation for which the estimated residual is above -1,496.58 (the negative of his estimated conduct coefficient) as being "impacted." **Figure A.2** demonstrates this fact. Using Dr. Singer's turnover materials, I confirm that the exact observations flagged by Dr. Singer as "impacted" are simply the observations with estimated residuals greater than -1,496.58.

[10] Wooldridge (2020), p. 74; Penn State Eberly College of Science Department of Statistics, "4.2 - Residuals vs. Fits Plot," available at https://online.stat.psu.edu/stat462/node/117.

[11] Assuming that the actual EIP for each student-school-year had a 50 percent chance of being higher than the predicted, but-for EIP in a setting with zero overcharge and that each proposed Class member had four observations, there is a $1-.5^4 = 93.75$ percent chance that at least one of a proposed Class member's actual EIPs would be above their predicted, but-for EIP.

[12] This example is chosen for illustrative purposes and other school/year combinations show the same general pattern.

Figure A.1: Residuals Estimated by Dr. Singer's Model for Proposed Class Member-Year Observations Enrolled at Cornell in 2022-2023[13]

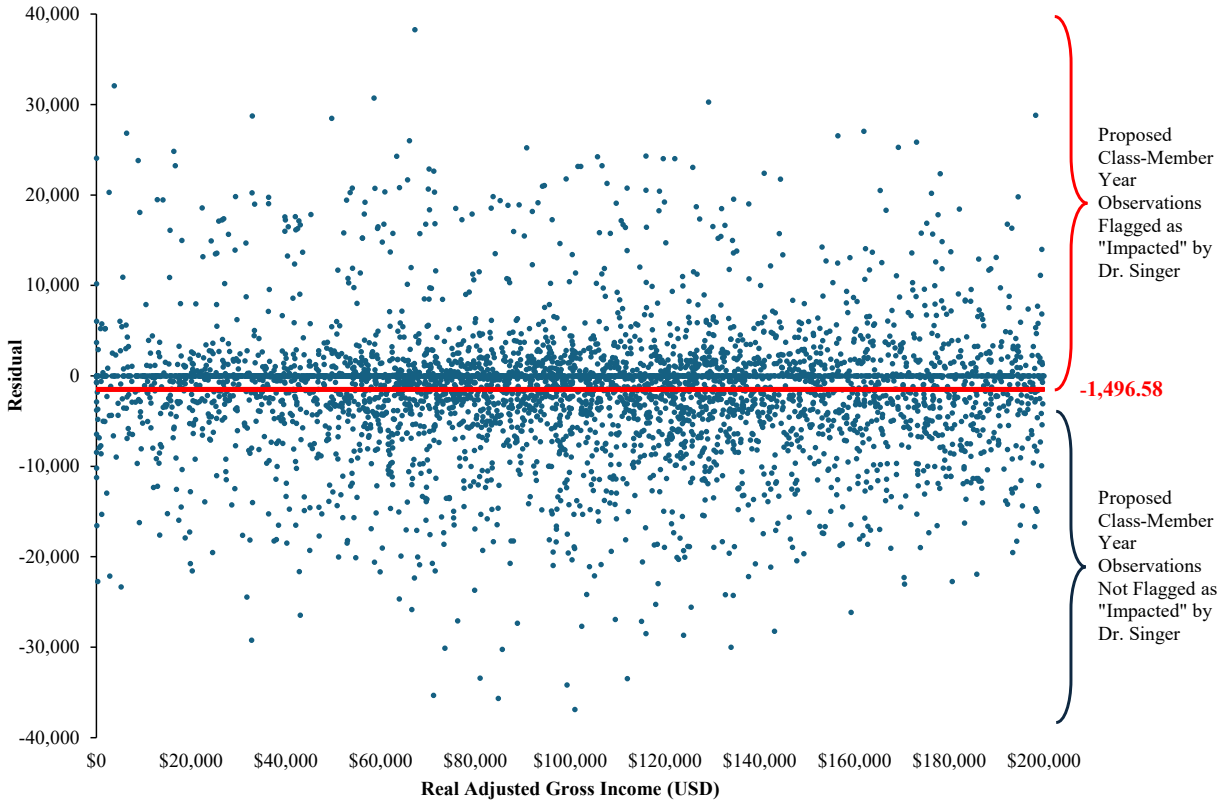

Figure A.2: Proposed Class Member-Year Observations Flagged as Impacted by Dr. Singer Compared with Those with a Residual Greater than -1,496.58[14]

Criteria	Class Member-Year Observations (Count)	Percent of Total Observations (Percent)
(a)	(b)	(c)
Flagged as Impacted in Dr. Singer's Turnover	367,709	65 %
Residual Greater than -1,496.58	367,709	65

10. To the extent that individual variation in injury from the Challenged Conduct is present, it is subsumed into the residuals obtained through Dr. Singer's methodology. Suppose for the sake of argument that Dr. Singer has correctly estimated the average impact of the Challenged Conduct. Because Dr. Singer has hardwired a uniform, average impact into the regression model he uses as the basis for his analysis, if there are individualized, specific deviations from this average, these differences get baked into the estimated

[13] Singer turnover materials (Defendants FA Regression Data).

[14] Singer Amended Report, Table 11; Singer turnover materials (Defendants FA Regression Data).

residuals. More generally, it is not possible to know what precisely is driving the unexplained variation leftover in a regression model. Dr. Singer's full model predicts EIPs that deviate substantially from EIPs actually paid. **Figure A.3** provides counts and percentages of observations with estimated residuals from Dr. Singer's model above various thresholds. Again, these estimated residuals are obtained from Dr. Singer's regression model which incorporates an average coefficient for impact (*i.e.*, these represent any remaining unexplained variation in price not accounted for in the model as specified). For example, for 34 percent of observations, Dr. Singer's model estimates a residual that is greater than the value of his asserted, average overcharge.

Figure A.3: Proposed Class Member-Year Observations With Residuals of Varying Magnitudes[15]

Criteria	Class Member-Year Observations	Percent of Total Observations
	--------(Count)--------	----(Percent)-----
(a)	(b)	(c)
Residual Greater than 0	246,954	43 %
Residual Greater than 1,496.58	191,502	34
Residual Greater than 10,000	50,291	9

11. Dr. Singer purports to use the estimated coefficients from his model to predict a but-for price for each student-school-year observation if the estimated conduct coefficient were set to zero, and claims that any proposed Class Member who had an observed EIP higher than the predicted but-for price in any academic year during the Conduct period was harmed by the Challenged Conduct. However, this approach ignores the inherent uncertainty in statistical estimation. Each of the coefficients in Dr. Singer's regression analysis are estimated with error, and so too are the resulting predictions of price using those coefficients. Analysts typically account for this known lack of precision in forecasting by building confidence intervals (*i.e.*, a plus-minus range) around an estimate. Dr. Singer ignores this known uncertainty in his predicted but-for EIP estimates and his opinions regarding the number of class members identified as impacted.[16] Putting aside the issues with Dr. Singer's methodology discussed above and elsewhere in my report, a proposed Class member whose actual EIP falls within a standard prediction interval around the estimated EIP for that proposed Class member should not be counted as impacted even in Dr. Singer's methodology, because some variation around a mean is expected. Without accounting for this known uncertainty and lack of precision in statistical estimation, Dr. Singer overstates the share of the class that he opines is harmed by the Challenged Conduct even if his model were otherwise capable of estimating

[15] Singer Amended Report, Table 11; Singer turnover materials (Defendants FA Regression Data).

[16] This uncertainty is known as "prediction error" (*see, e.g.*, Wooldridge (2020), pp. 201-204). The standard error of the predicted price can be used to obtain a 95 percent confidence interval for each of the purported but-for price estimates. A 95 percent confidence interval is the range of values for which there is a 95 percent chance that the true value of the point estimate is within that range, assuming Dr. Singer's regression model is correctly specified.

impact from the Challenged Conduct, which it is not for the reasons described in my report.